

Evidence for quenched chiral logs*

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Using the pole shifting procedure of the modified quenched approximation (MQA) to cure the exceptional configuration problem, accurate hadron spectrum calculations can be obtained at very light quark mass. Here we use the MQA to extend and improve our previous investigation of chiral logs in the pion mass. At $\beta = 5.7$ for Wilson fermions, we see clear evidence for quenched chiral logarithms in the pion mass as a function of quark mass. The size of the observed chiral log exponent δ is in good agreement with the value obtained from a direct calculation of the η' hairpin diagram.

1. INTRODUCTION

Some time ago Sharpe and Bernard and Goltermann[1] showed that the quark mass dependence of certain physical quantities near the chiral limit should serve as a particularly incisive test of the difference between quenched and full QCD. In particular, the behavior of the pion mass as a function of the quark mass in quenched QCD is expected to be non-analytic at $m_q = 0$, with an anomalous exponent δ ,

$$m_\pi^2 \propto (m_q)^{\frac{1}{1+\delta}} \quad (1)$$

The value of the chiral log exponent in the pion mass should be related to the coefficient of the η' hairpin insertion by

$$\delta = \frac{m_0^2}{48\pi^2 f_\pi^2} \quad (2)$$

Calculations we have reported previously[2] showed no evidence for chiral logs in the pion mass at $\beta = 5.7$ over the range of hopping parameters used in that calculation, from .1610 to .1680 (pion masses of .650 to .254 in lattice units). The value of the exponent $\delta = .015(47)$ was obtained, i.e. consistent with zero. The results reported here are completely consistent with the previous

negative results. We find that the value of δ obtained in this region from the hairpin coefficient varies from $\delta < .01$ at $\kappa = .161$ to $\delta = .045(5)$ at $\kappa = .168$. Even when extrapolated to the chiral limit, the exponent is only $\delta = .053(7)$.

In this present calculation, we have used the pole-shifting ansatz of the modified quenched approximation (MQA) [3] to resolve the exceptional configuration problem. This allows us to greatly improve our statistics at light quark masses and to extend the calculations much closer to critical hopping parameter. The pion mass at these lighter quark mass values is much more sensitive to the chiral logs and our errors are small enough to see a significant effect even though $\delta \approx .05$. At the lightest three quark masses (.1683, .1685, and .1687) we find clear evidence for a deviation from linearity at just the level and functional form expected from the hairpin calculation. The agreement between the hairpin result and the pion mass behavior provides significant numerical support for the theoretical picture of chiral logarithms and their connection to η' loops in the quenched approximation.

Both the overall smallness of the chiral log parameter and the strong quark mass dependence are somewhat unexpected, in light of the fact that (a) the physical η' mass gives $\delta \approx .17$, and (b) in the chiral limit, the hairpin mass insertion is related by the Witten-Veneziano formula to

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Table 1
 η' mass m_0 , predicted chiral log parameter δ , and pion mass.

κ	m_0	δ	m_π
.1687			.165(5)
.1685	.383(22)	.050(6)	.195(4)
.1683			.221(3)
.1680	.371(20)	.045(5)	.254(2)
.1675	.346(16)	.038(3)	.298(2)
.1667	.316(14)	.030(3)	.356(2)
.1650	.268(13)	.019(2)	.458(1)
.1630	.221(13)	.011(1)	.558(1)

the topological susceptibility of pure glue (without quarks) and should therefore be rather insensitive to the quark mass (at least for quark masses much smaller than the QCD scale). In an effort to understand the origin of this suppression and quark mass dependence, we carried out another calculation of the hairpin diagram using a clover improved quark action. (The clover coefficient used here is $C_{sw} = 1.57$.) The two main effects of clover improvement are (1) to increase the hairpin coefficient m_0^2 by more than a factor of 2, and (2) to decrease the slope of m_0 as a function of quark mass by about a factor of 2 (where quark mass for the clover case is rescaled so that equal quark mass corresponds to equal pion mass). This strongly suggests that both the suppression and the strong quark mass dependence of m_0 are lattice artifacts. Thus we would expect the chiral log parameter δ to be quite sensitive to lattice spacing and to increase substantially at higher β values. Recent results on the pion mass from the *CP-PACS* collaboration[4] indicating that $\delta \approx .1$ at $\beta = 6.47$ are quite consistent with this conclusion.[5]

In Table I the results for m_0 are presented. Also listed are the resulting values of the chiral log exponent δ , using Eq. (2), with f_π calculated on the same ensemble. In the last column, we list the values of the pion mass obtained from the valence pion propagator.

As a function of κ^{-1} , the values of m_π^2 obtained from Table I are quite linear for the heavier masses, but show a significant deviation from lin-

earity for $\kappa > .1680$. This deviation has both the form and the magnitude predicted by the chiral log formula (1) combined with the hairpin values for δ . A direct fit of m_π^2 to the formula (1), including values from $\kappa = .165$ to $\kappa = .1687$ gives $\delta = .058(39)$, i.e. significantly different from zero and consistent with the hairpin results. However, this direct fitting procedure with constant δ has several drawbacks. The rather large error reflects the observed fact that the value of δ changes significantly over the fit range. Also, the value of κ_c must be taken as one of the fitting parameters, and there is a strong correlation between κ_c and δ .

A more accurate way to observe the presence and magnitude of the chiral logs is to consider the ratio of differences:

$$\frac{\Delta(m_\pi^{2(1+\delta)})}{\Delta(\kappa^{-1})} \quad (3)$$

where the differences are computed between adjacent values of κ . With the correct choice of δ , this ratio should be constant. Note that calculating this ratio does not require a determination of κ_c . Another advantage is that the values of m_π at different κ 's are positively correlated, so that errors on the difference are typically 30-50% smaller than on the (mass)² itself. The errors are computed by a single elimination jackknife. To see the chiral log effect, we first plot in Fig. 1 the ratio (3) for $\delta = 0$. The horizontal dashed line represents the result of the best linear fit to m_π^2 for the κ range .163 to .168. The three points at the lightest quark mass, < 25 MeV, are obtained from the three differences computed between the four kappa values .1680, .1683, .1685, and .1687. These points show a significant chiral log effect which is consistent with the hairpin expectations. To see this we plot in Fig. 2 the ratio (3) with values of δ taken from interpolating the hairpin results in Table I. With this choice of δ , the ratio's are consistent with a constant. To see how sensitive this plot is to the chosen values of δ , we plot in Fig. 3 the ratios (3) with a mass independent value of $\delta = .1$. The behavior of the ratio is now completely inconsistent with a constant, illustrating that a value as large as $\delta = .1$ is ruled out by the data.

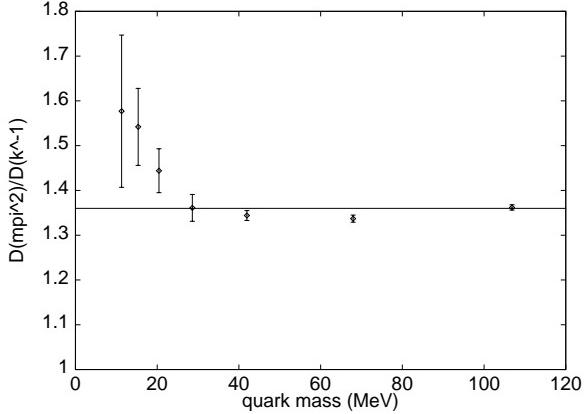


Figure 1. The ratio $\Delta(m_\pi^2)/\Delta(\kappa^{-1})$, i.e. Eq. 3 with $\delta = 0$.

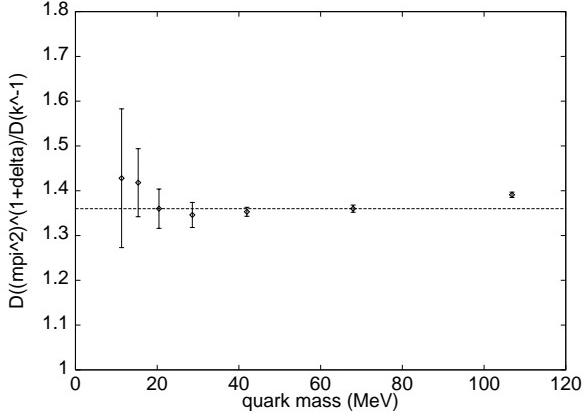


Figure 2. The ratio $\Delta(m_\pi^{2(1+\delta)})/\Delta(\kappa^{-1})$ with δ from Table I.

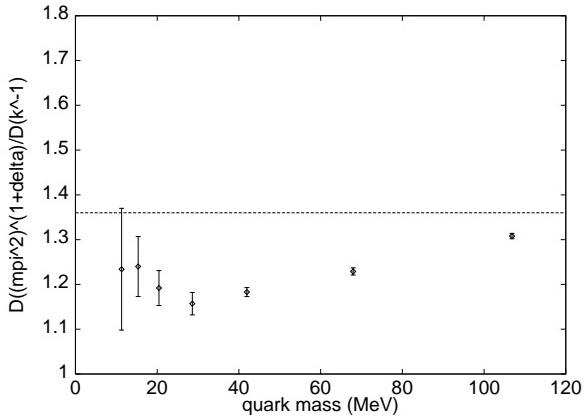


Figure 3. The ratio $\Delta(m_\pi^{2(1+\delta)})/\Delta(\kappa^{-1})$ with $\delta = 0.1$.

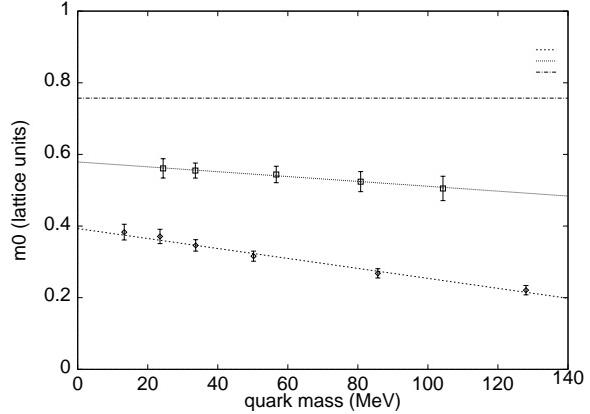


Figure 4. Comparison of m_0 for Wilson action (lower line) and clover action with $C_{sw} = 1.57$ (middle line). The upper line is the result of a linear extrapolation to zero slope.

Finally, we present in Fig. 4 the values of m_0 (in lattice units) obtained from the clover improved action, compared with those obtained from the unimproved Wilson action. As discussed above, the clover improved value is substantially larger in magnitude and the quark mass dependence is significantly reduced.[5]

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